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## JMO Number Theory Questions

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**Level: Junior    Ref No: J01**

**Puzz Points: 10**

A number like 4679 is called an ascending number because each digit in the number is larger than the preceding one.

- (i)     How many ascending numbers are there between 1000 and 2000?
- (ii)    How many ascending numbers are there between 1000 and 10000?

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**Level: Junior    Ref No: J10**

**Puzz Points: 13**

How many different solutions are there to the letter sum on the right? Different letters stand for different digits, and no number begins with a zero.

$$\begin{array}{r} J M C \\ + J M O \\ \hline S U M S \end{array}$$

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**Level: Junior    Ref No: J11**

**Puzz Points: 14**

- (i)     Explain why the sum of three consecutive integers is always divisible by 3.
- (ii)    Is it true that the sum of four consecutive integers is always divisible by 4?
- (iii)   For which  $k$  is it true that the sum of  $k$  consecutive integers is always divisible by  $k$ ?

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**Level: Junior    Ref No: J13**

**Puzz Points: 10**

Find four integers whose sum is 400 and such that the first integer is equal to twice the second integer, three times the third integer and four times the fourth integer.

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**Level: Junior    Ref No: J16**

**Puzz Points: 13**

Find a rule which predicts exactly when five consecutive integers have sum divisible by 15.

**Level: Junior Ref No: J19**

**Puzz Points: 10**

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In her purse, Jenny has 20 coins, with a total value of £5. There are three denominations of coin – 10p, 20p and 50p – in her purse and she has more 50p coins than 10p coins. How many of each type of coin does she have?

**Level: Junior Ref No: J20**

**Puzz Points: 11**

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$$97 \rightarrow 63 \rightarrow 18 \rightarrow 8$$

An example of a particular type of number chain is shown above. The first number must be a positive integer. Each number after the first is the product of the digits of the previous number, so in this case  $63 = 9 \times 7$ ;  $18 = 6 \times 3$ ;  $8 = 1 \times 8$ . The chain stops when a single-digit number is reached.

Suppose that in such a chain the final number is 6. Find all possible two-digit first numbers for this chain.

**Level: Junior Ref No: J29**

**Puzz Points: 14**

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Observe that  $49 = 4 \times 9 + 4 + 9$

- (i) Find all other two-digit numbers which are equal to the product of their digits plus the sum of their digits.
- (ii) Prove that there are no three-digit numbers which are equal to the product of their digits plus the sum of their digits.

**Level: Junior Ref No: J31**

**Puzz Points: 10**

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Every digit of a given positive integer is either a 3 or a 4 with each occurring at least once. The integer is divisible by both 3 and 4.

What is the smallest such integer?

**Level: Junior Ref No: J33**

**Puzz Points: 12**

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When Dad gave out the pocket money, Amy received twice as much as her first brother, three times as much as the second, four times as much as the third and five times as much as the last brother. Peter complained that he had received 30p less than Tom.

Use this information to find all the possible amounts of money that Amy could have received.

**Level: Junior Ref No: J40**

**Puzz Points: 13**

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For each positive two-digit number, Jack subtracts the units digit from the tens digit; for example, the number 34 gives  $3 - 4 = -1$ . What is the sum of all his results?

(Note: to get credit for this question, you need to give a moderately concise reason for your answer, rather than simply listing out every single 2-digit number)

**Level: Junior Ref No: J50**

**Puzz Points: 11**

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The eight-digit number " $ppppqqqq$ ", where  $p$  and  $q$  are digits, is a multiple of 45.

What are the possible values of  $p$ ?

**Level: Junior Ref No: J55**

**Puzz Points: 10**

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In 2007 Alphonse grew twice the number of grapes that Pierre did. In 2008 Pierre grew twice the number of grapes that Alphonse did. Over the two years Alphonse grew 49 000 grapes, which was 7600 less than Pierre. How many grapes did Alphonse grow in 2007?

**Level: Junior Ref No: J60**

**Puzz Points: 15**

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I want to choose a list of  $n$  different numbers from the first 20 positive integers so that no two of my numbers differ by 5. What is the largest value of  $n$  for which this is possible? How many different lists are there with this many numbers?